

Multinucleon ejection model for Meson Exchange Current neutrino interactions

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A model is proposed to describe pairs or triples of nucleons ejected from nucleus as a result of Meson Exchange Current neutrino interaction. The model can be easily implemented in Monte Carlo neutrino event generators. It can provide a help in identifying true charge current quasielastic events and allow for better determination of the systematic error of neutrino energy reconstruction in neutrino oscillation experiments.

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I. INTRODUCTION

There is a lot of evidence for a significant multinucleon ejection contribution to the inclusive neutrino charge current (CC) cross section in the 1 GeV energy region¹. On the experimental side, several recent nuclear target CCQE (CC quasi elastic) cross section measurements reported large values of the axial mass in disagreement both with older deuterium data and also with independent electroproduction arguments². An explanation for the discrepancy would be that some events interpreted as CCQE are in fact due to very different dynamical mechanism which typically leads to multinucleon emission. In fact, in the MiniBooNE (MB) experiment nucleons in the final state were not analyzed and only events with pions were identified and rejected. MB collaboration reported high statistics 2-dimensional muon inclusive cross section which is likely to include a large multinucleon ejection contribution and is a challenge for theoretical models³.

On the theoretical side they are several computations which support the idea of the large multinucleon contribution. In the context of neutrino interactions the first was the Marteau model, based on the earlier ideas of Ericsson and Delorme⁴. The model uses the non-relativistic Fermi Gas in the Local Density Approximation (LDA) approach. It includes elementary QE and Δ excitation interactions, Random Phase Approximation (RPA) in medium polarization effects important in the low Q^2 region⁵ and Δ width modification in the nuclear matter⁶. The model was later upgraded by Martini, Ericson, Chanfray and Marteau (MEChM model)⁷. With the inclusion of relativistic corrections MB 2D differential cross section can be reproduced⁸.

Another approach to multinucleon ejection was proposed by Nieves, Ruiz Simo and Vicente Vacas⁹. The model is based on the earlier computations of the inclusive electron-nucleus cross section in the energy region of QE and Δ excitation peaks and also in the so called dip region between them¹⁰. In the paper¹¹ the fit was done

to the MB 2D data with the best fit axial mass value equal to 1.08 ± 0.03 GeV.

Both above mentioned approaches are non-relativistic although several relativistic corrections were introduced based on the authors experience from dealing with the electron scattering data.

An attempt to include relativistic effects in a more systematic way was undertaken in a series of papers¹². A 2p-2h (two particles and two holes) contribution to both transverse and longitudinal responses coming from Meson Exchange Current (MEC) and correlation diagrams were evaluated. Under some assumptions a relativistic MEC contribution to the neutrino inclusive cross section was also evaluated.

The results of more rigorous computations done for electron scattering on lighter nuclei¹³ show that one can expect a large contribution to the transverse response coming from two-body current. But it seems to be important to have a more realistic description of the nucleus ground state than it was assumed in all the previously mentioned papers.

The situation is little confusing as the models which have better description of the ground state, like the Spectral Function approach¹⁴, do not lead to better agreement with the MB data unless again a very large value of the axial mass is assumed¹⁵. It seems that it is important to have both Short Range Correlations (SRC) and MEC effects in the same relativistic computational scheme which is however very difficult to achieve.

Recently still one other effective approach to describe multinucleon contribution to the neutrino inclusive cross section was proposed. The Transverse Enhancement Model (TEM)¹⁶ is based on the analysis of the electron-carbon scattering data and parameterizes the MEC effect as a modification of the magnetic electromagnetic form factor. The model predicts that the MEC contribution is less important at larger neutrino energies which can reconcile MB and NOMAD¹⁷ axial mass measurements.

It is clear that understanding of the multinucleon ejection contribution is required for neutrino energy reconstruction which is in turn crucial for correct interpretations of high precision neutrino oscillation measurements. It is important to have a model of MEC contribution

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which can be implemented in the neutrino Monte Carlo (MC) event generators¹⁸ and for that one needs also predictions for the nucleons in the final state. With such a model one can start a direct comparison between what is measured and what is predicted in order at least of evaluate the systematic error due to a lack of more precise multinucleon ejection model. Theoretical models provide only differential cross sections in the muon three momentum and a model of multinucleon ejection is something with which the theoretical models must be supplemented. Having in mind an importance of the predictions for the final state nucleons we propose a simple model to provide such information which can be confronted with the data.

II. INCLUSIVE MUON CROSS SECTION MODELS

The multinucleon ejection model of this paper is based on the information contained in the muon inclusive differential cross section only. If the kinematic variables are chosen to be energy and momentum transfer, one knows their joint probability distribution as a function of neutrino energy. We will propose a procedure to predict nucleon momenta in the final state.

We will discuss two models for which it is impossible to get exact predictions for the final hadronic system and one must rely on approximate modeling like the one we are discussing in this paper.

A. Simplified Marteau (SM) model

The first model we consider is a simplified version of the Marteau model. It was developed about 10 years ago and it includes RPA, Δ width, but not LDA effects. Its basic ingredients are presented in the reference¹⁹. Recently, elementary 2p-2h responses were added using the procedures described in⁷.

Fig. 1 show the comparison between predictions for the total cross section of the SM model with its full version. Here the term *CCQE* means that only one nucleon is ejected from nucleus. The SM model predicts larger cross section for the CCQE and lower for np-nh. Fig. 2 shows that for larger neutrino energies a part of the difference for QE is due to the fact that the SM model is more relativistic than the MEChM model. This is because in the computations the relativistic Lindhard function was used.

The SM model predicts (like MEChM) both two- and three- nucleon ejection. The three nucleon contribution comes from the pionless Δ decay i.e. from reactions $N\Delta \rightarrow NN$ and $NN\Delta \rightarrow NNN$ with N standing for a nucleon⁶.

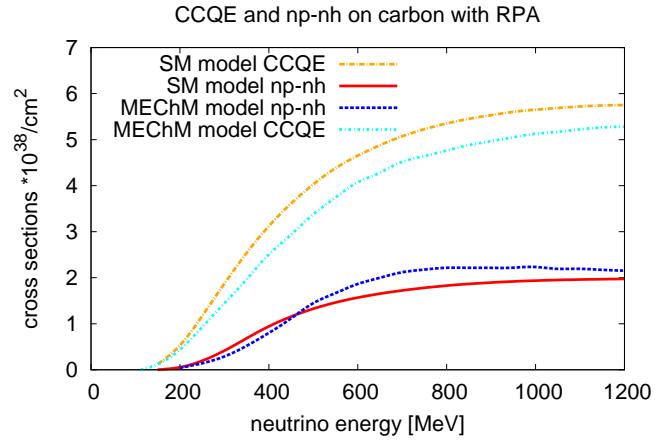


FIG. 1. A comparison of predictions of the simplified Marteau (SM) model used in this paper and the *full* model called here MEChM. The target is carbon and all the nuclear effects are included. We show predictions for QE events (one nucleon ejection) and multinucleon (two or three) ejection. For MEChM the points are taken from⁷.

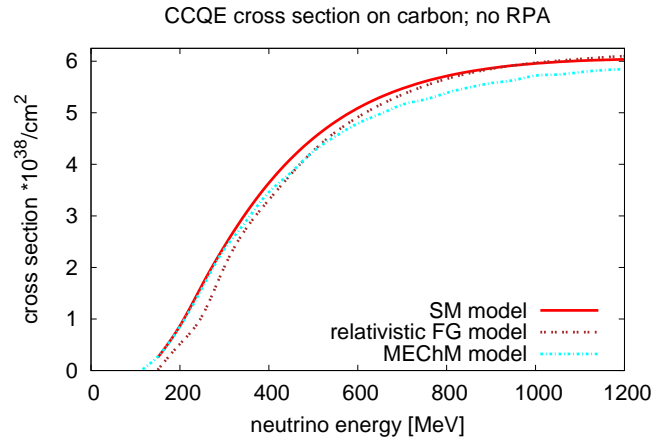


FIG. 2. A comparison of predictions for QE events *before* RPA nuclear effects. We show predictions of the SM model, MEChM model and also relativistic FG model. For MEChM the points are taken from⁷.

B. Transverse Enhancement Model (TEM)

In¹⁶ a new approach to describe CCQE scattering on nuclear targets is proposed. The model is easy to implement in MC event generators. It is sufficient to modify vector magnetic form factors keeping all other ingredients of the CCQE model as in the free nucleon target case.

The authors of¹⁶ proposed a universal transverse enhancement function of Q^2 for the carbon target. For low Q^2 its form is determined by the scaling arguments while for high Q^2 ($> 0.5 \text{ GeV}^2$) it is obtained as a fit to the inclusive electron cross section data from the JUPITER experiment. The prescription to include transverse enhancement contribution in the numerical computations

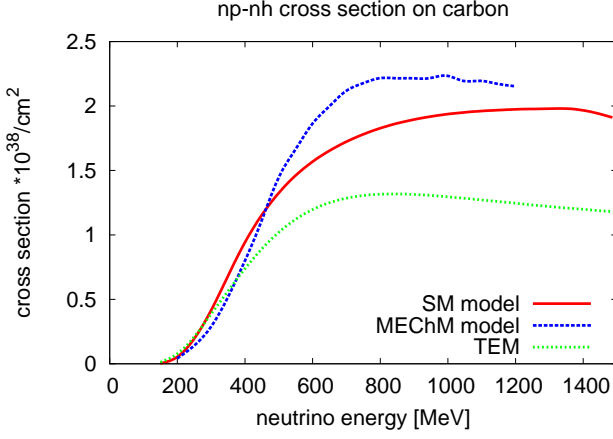


FIG. 3. A comparison of predictions for the total multinucleon ejection cross section from three models: SM model, TEM and MEChM model. The target is carbon.

amounts to the replacement:

$$G_M^{p,n}(Q^2) \rightarrow \tilde{G}_M^{p,n}(Q^2) = \sqrt{1 + A Q^2 \exp(-\frac{Q^2}{B})} G_M^{p,n}(Q^2) \quad (1)$$

where $G_M^{p,n}(Q^2)$ are electromagnetic form-factors, $A = 6 \text{ GeV}^{-2}$ and $B = 0.34 \text{ GeV}^2$.

The most interesting feature of the TEM model is that it offers a possible explanation to the apparent contradiction between low (MB) and high (NOMAD) neutrino energy M_A measurements: for energies up to $\sim 700 \text{ MeV}$ TEM predicts the CCQE cross section to be similar to CCQE with $M_A = 1.3 \text{ GeV}$. For higher neutrino energies the TEM cross section becomes significantly smaller and at $E_\nu \sim 5 \text{ GeV}$ it corresponds to CCQE with $M_A \sim 1.15 \text{ GeV}$.

As the TEM prediction for the MEC contribution we simply take the difference between the cross sections calculated with modified and standard magnetic form factors:

$$\begin{aligned} \frac{d^2\sigma^{TEM}}{dq d\omega} &\equiv \frac{d^2\sigma^{CCQE}}{dq d\omega}(\tilde{G}_M^{p,n}) \\ &- \frac{d^2\sigma^{CCQE}}{dq d\omega}(G_M^{p,n}). \end{aligned} \quad (2)$$

In the case of TEM model we assume that only two nucleon ejection takes place.

C. Model comparison

Fig. 3 shows the contributions to the overall cross section on the carbon target coming from multinucleus ejection. There is a significant difference in the predicted

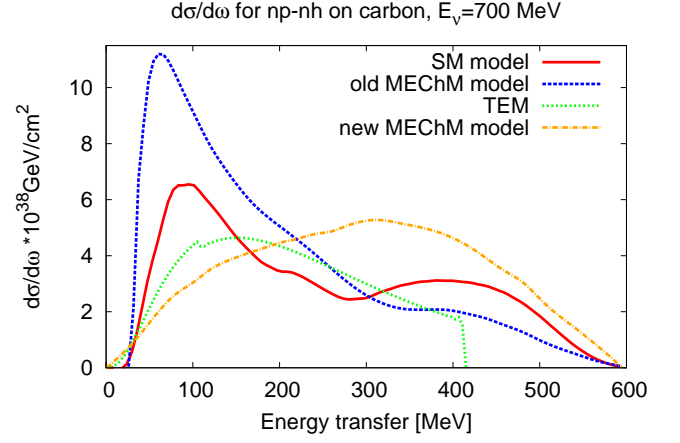


FIG. 4. A comparison of predictions for the differential cross section in energy transfer for multinucleon ejection from three models: SM model, TEM and MEChM model. In the case of MEChM model the results from old and new versions are shown separately.

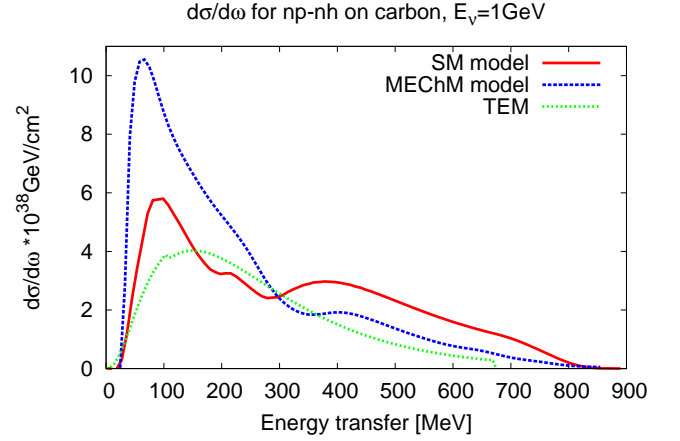


FIG. 5. As in the previous figure, with different neutrino energy. For 1 GeV the predictions from old MEChM model are not available.

size of the effect. It is an important feature of the TEM model that for larger neutrino energies the MEC contribution becomes relatively small.

Figs 4 and 5 show differential cross section for the multinucleon ejection contribution as a function of energy transfer for two values of the neutrino energy: 700 MeV and 1 GeV. We show also the available results from the MEChM model. Note, that there are two versions of the MEChM model with different elementary 2p-2h response functions. We followed the procedures described in⁷ and obtained new responses as functions of the variable $x \equiv Q^2/2M\omega = (q^2 - \omega^2)/2M\omega$ where ω and q denote energy and momentum transfers and M is nucleon's mass. However, the new response functions lead to rapidly increasing multinucleon ejection cross section as a function of neutrino energy. The behavior seems to

be unphysical and in this paper we rely on the original Marteau responses. In Figs 4 and 5 one can see significant differences between the models which will translate into predictions for the multinucleus ejection. The old MEChM model predicts the pronounced peak at low energy transfer which is probably non-physical and comes only from oversimplified assumptions for the response function. The SM model predicts the peak which is smaller in size and for larger values of the energy transfer develops a shape which is close to the new version of the MEChM model.

In the case of TEM the sharp fall down of the differential cross section at ~ 410 MeV comes from the way in which the model was implemented. We followed the original paper and assumed the target nucleon to be at rest. Pauli blocking effects are introduced by means of the Q^2 depending suppression function.

III. NUCLEON EJECTION MODEL

Two basic assumptions of the model is that the energy and momentum are transferred to two (or three) nucleons simultaneously and that there are no correlations between initial state nucleons. The second assumption can appear to be an unrealistic but its impact on the final state predictions is rather mild.

The scheme of the procedure to generate nucleon final state is as follows:

- two (or three) nucleons are selected from the Fermi sphere of radius 220 MeV (it is assumed that interaction occurs on carbon)
- four momentum of the hadronic system is calculated by adding four momenta of selected nucleons and energy and momentum transferred by the interacting neutrino
- Lorentz boost to the hadronic center of mass system is done
- two (or three) nucleons are selected isotropically in the hadronic center of mass system
- boost back to the laboratory frame is performed.

The energy balance is done based on the assumptions that initial state nucleons are in the potential well of the depth $V = E_f + 8$ MeV (E_f is the Fermi energy).

- Fermi energy is subtracted from each initial state nucleon
- for each nucleon in the final state (in the LAB frame) the energy is reduced by the amount of 8 MeV adjusting its momentum so that they remains on-shell.

The above procedure allows for a smooth distribution of nucleon momenta in the final state.

Pauli blocking was not imposed for two reasons. First of all, the final nucleon state is calculated for events about which it is known that they occur and a reduction of the cross section is not required. Secondly, a smooth spectrum of final state nucleons should be obtained. Clearly, other prescriptions to account for the energy balance can be invented but the above procedure seems to be the simplest one and satisfying basic physical requirements.

It is an interesting observation that the above described algorithm actually introduces some correlations between initial state nucleons. Not all initial state nucleon configurations give rise a hadronic system which in the center of mass frame has the invariant mass larger than $2M$ (or $3M$ for three nucleon ejection; M is nucleon's mass). In the numerical code initial state nucleon configurations are selected until acceptable nucleon momenta are found.

We investigated an impact on the model predictions if a pair of nucleons in the initial state is chosen to be always in the deuteron-like configuration i.e. with opposite three-momenta. It turned out that the results are virtually unchanged. The most important assumption of the model is that of the isotropic distribution of final nucleons in the hadronic center of mass frame.

Clearly, in the MC event generator there should also be included a contribution from SRC large momentum nucleon pairs. This contribution can be obtained in the SF formalism with a straightforward MC implementation (a simpler alternative is the Bodek-Ritchie model²⁰). In the SF approach the lepton interacts with only one nucleon which absorbs all the energy and momentum that is transferred to the hadronic system. The other nucleon, with approximately opposite initial three momentum is a spectator only.

In the MC simulation all the above described steps should be followed by Final State Interactions (FSI).

IV. RESULTS

The series of figures show the comparison of predictions from both models for the inclusive muon cross section (TEM and SM model) with the same model of multinucleon emission. All the distributions are normalized to the same area so that only their shapes are compared.

A. Total kinetic energy

Figs 6 and 7 show the total kinetic energy of all the nucleons in the final state. Even if individual nucleons cannot have reconstructed tracks due to too small momenta, the total kinetic energy can be measured as a vertex activity. One can see that the predictions from two models differ significantly. The SM model predicts

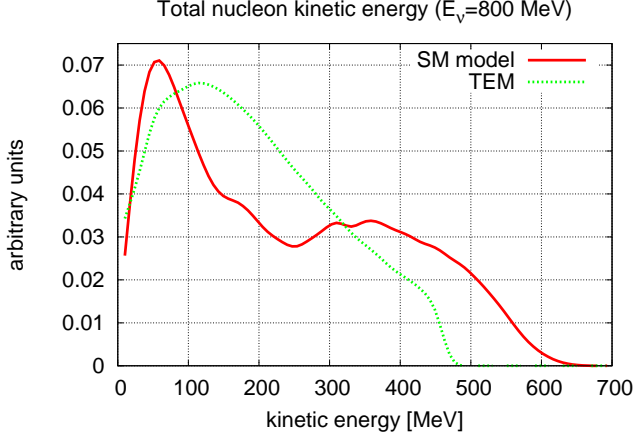


FIG. 6. Total kinetic energy of the final state nucleons. Neutrino energy is 800 MeV.

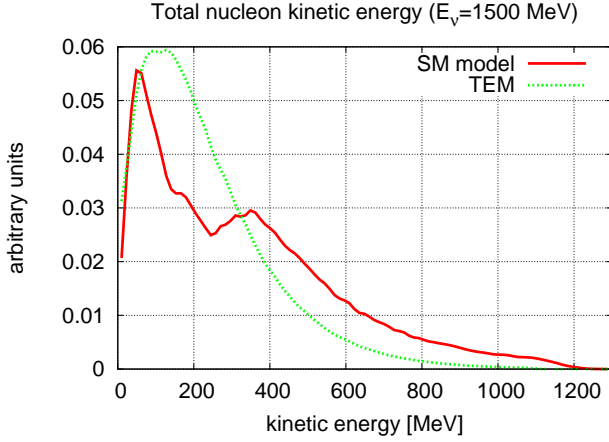


FIG. 7. Total kinetic energy of the final state nucleons. Neutrino energy is 1500 MeV.

more events with large energy deposit. In order to understand this one should look back at the Figs 4 and 5: TEM predicts much smaller cross section at large values of the energy transfer which translates to the total kinetic energy of the ejected nucleons.

In the case of the SM model we would like also to show separate contributions from two- and three- nucleon final states, see Fig. 8. It can be seen that three nucleon ejection contribute significantly to the maximum at larger values of the kinetic energy.

B. Nucleon momenta

Figs 9 and 10 show the distributions of the momenta of most energetic nucleon in the final state. This is an important prediction because it can tell us how likely it is that a MEC event will have a nucleon above the detection threshold. The main difference between TEM

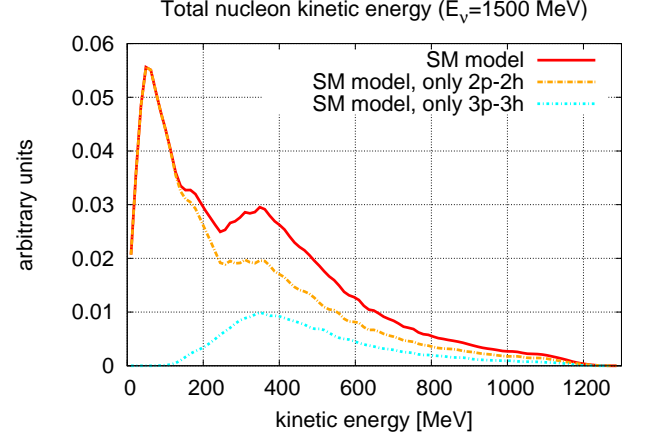


FIG. 8. Contributions from two- and three- nucleon final states to the overall SM model predictions for the total kinetic energy of ejected nucleons. Neutrino energy is 1500 MeV.

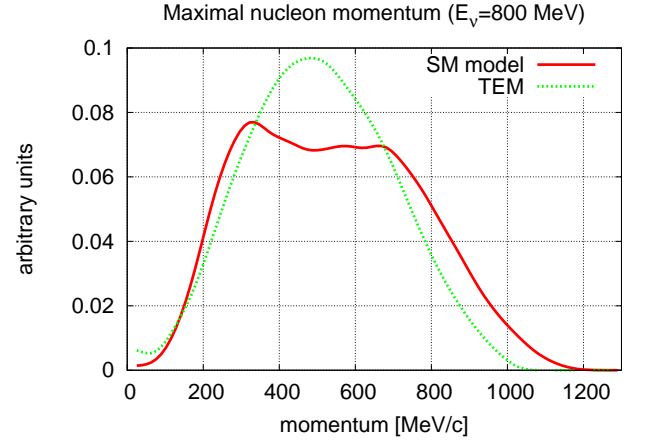


FIG. 9. Momentum of the most energetic nucleon in the final state. Neutrino energy is 800 MeV.

and SM model is that the first one predicts a maximum at the value ~ 500 MeV/c while the other model prediction have the peak cut off and the extended large momentum tail.

Figs 11 and 12 show the distributions of the momentum of the second most energetic nucleon in the final state. In the case of two nucleon ejection it is also the lowest momentum nucleon but in the case of the SM model there is also a fraction of events with three ejected nucleons. One can be surprised that the second nucleon can be also quite energetic.

C. Angular distribution

We calculated the cosine of the angle θ_{QE} between the most energetic nucleon momentum and the hypothetical three momentum of the nucleon if the interaction were

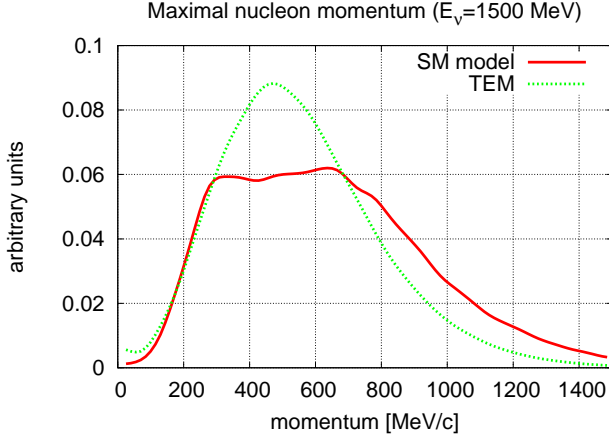


FIG. 10. Momentum of the most energetic nucleon in the final state. Neutrino energy is 800 MeV.

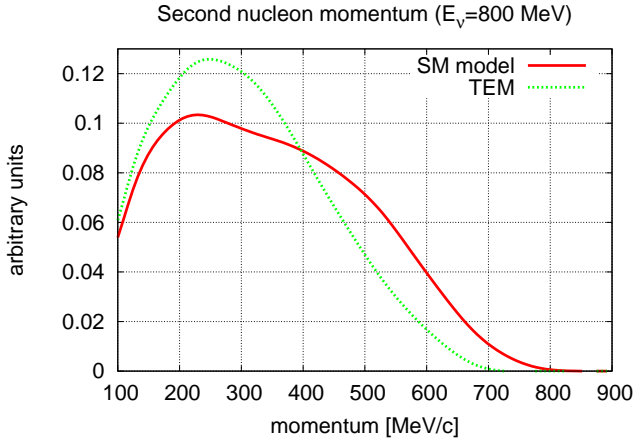


FIG. 11. Momentum of the second most energetic nucleon in the final state. Neutrino energy is 800 MeV.

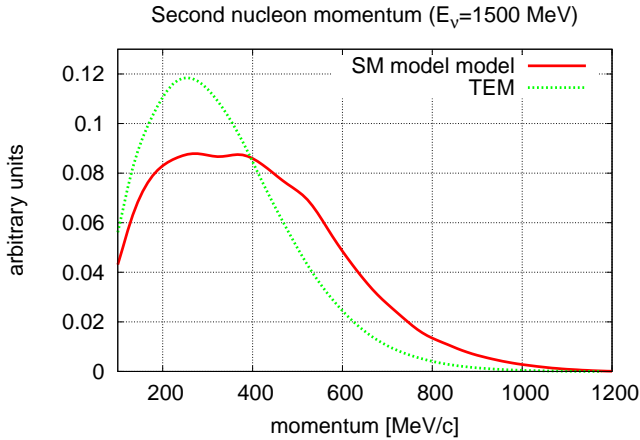


FIG. 12. Momentum of the second most energetic nucleon in the final state. Neutrino energy is 1500 MeV.

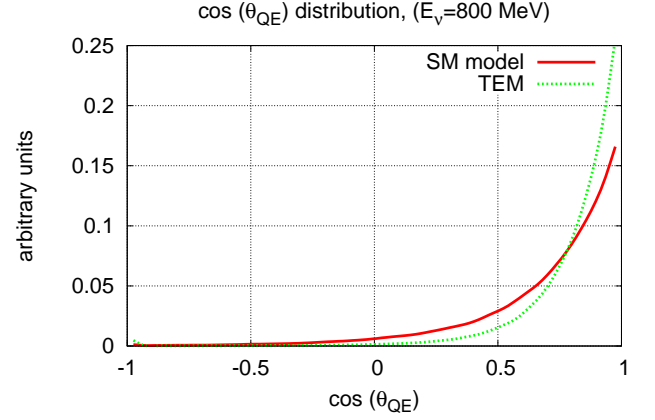


FIG. 13. Cosine of the angle between the most energetic nucleon three momentum and hypothetical three momentum of the nucleon if the interaction were CCQE and the target nucleon at rest. Neutrino energy is 800 MeV.

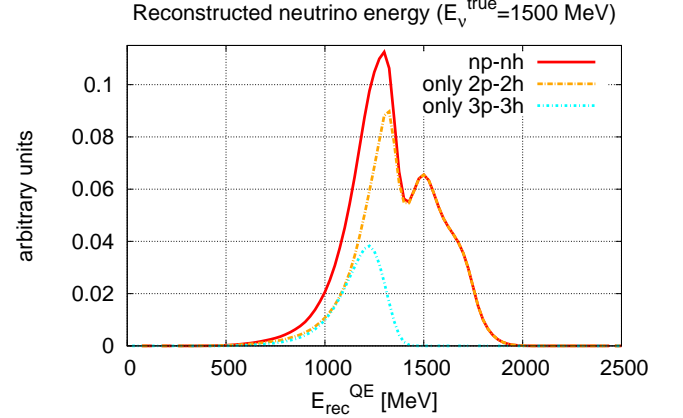


FIG. 14. Reconstructed neutrino energies for MEC interactions of $E_\nu = 1500$ MeV neutrino. Contributions from two and three nucleon ejection are also shown separately.

CCQE with the target nucleon at rest (θ_{QE} is calculated based only on muon's kinetic energy and production angle). In experimental analyses θ_{QE} is often used to select CCQE enriched samples of events with a criterium defined as $\theta_{QE} < 30^\circ$. Fig. 13 show that for neutrino energy 800 MeV the distribution is strongly forward peaked and the nucleons ejected due to MEC mechanism seem to *mimic* those which arise after genuine CCQE interaction. For neutrino energy 1500 MeV the results are very similar.

D. Energy reconstruction

Fig. 14 shows the distribution of E_{rec}^{QE} obtained within the SM model. The true neutrino energy is 1500 MeV.

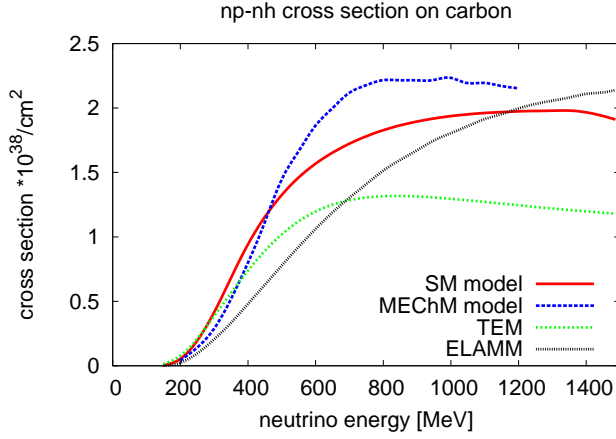


FIG. 15. ELAMM predictions for the total multinucleon ejection cross section is compared with: SM model, TEM and the MEChM model.

E_{rec}^{QE} is defined in the standard way by assuming that only final muon is detected, the interaction was CCQE and the target nucleon was at rest. Two peaks seen on the figure come from various ingredients of the SM model. The most important observation is that on average the values of E_{rec}^{QE} are much smaller than the true neutrino energy.

V. DISCUSSION

A. ELAMM

In order to get still better intuition how large uncertainties of the model can be, we would like to introduce still another effective MEC ‘model’, the one defined by the large axial mass itself! In the ‘model’ which we shall call ELAMM (Effective Large Axial Mass Model):

$$\frac{d^2\sigma^{MEC}}{dq d\omega} \equiv \frac{d^2\sigma^{CCQE}}{dq d\omega}(M_A = 1.35) - \frac{d^2\sigma^{CCQE}}{dq d\omega}(M_A = 1.05) \quad (3)$$

ELAMM has an important advantage that it reproduces very well the MB 2D differential cross section data.

Figs 15, 16 and 17 show that ELAMM predictions are quite different from the TEM and the SM model. It can be surprising that at the typical MB neutrino flux energies ~ 700 MeV the ELAMM multinucleus ejection cross section is much smaller than that of the SM model. Also ELAMM np-nh cross section increases for larger neutrino energies which is not the case for other models. It is rather clear that the way in which the ELAMM describes the excess of events due to MEC dynamics is rather different to the one provided by TEM and SM model.

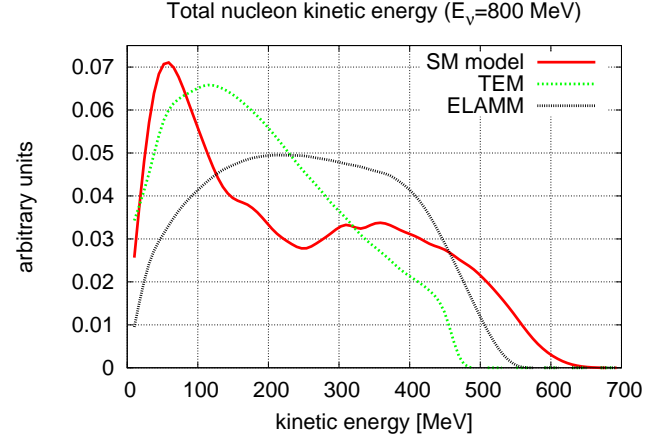


FIG. 16. ELAMM predictions for the total kinetic energy of the final state nucleons is compared with: TEM and SM model. Neutrino energy is 800 MeV.

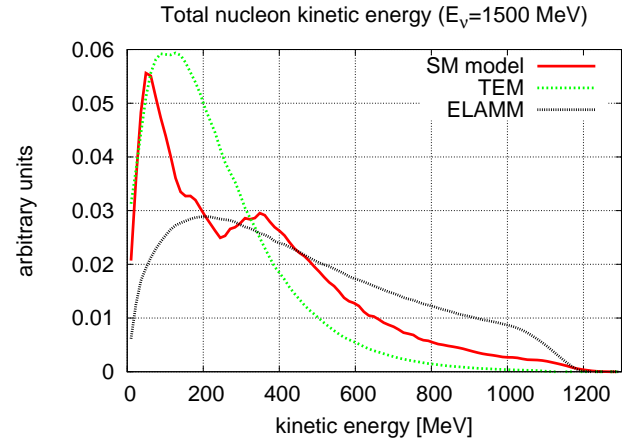


FIG. 17. ELAMM predictions for the total kinetic energy of the final state nucleons is compared with: TEM and SM model. Neutrino energy is 1500 MeV.

B. Isospin

The question which was avoided so far but which is very important from the experimental point of view (it is much easier to detect protons than neutrons) is that of isospin state of ejected nucleons.

In the case of SRC nucleon pairs it is known that it is much more likely to have proton-neutron pairs than neutron-neutron or proton-proton ones^{13, 22}. It is not obvious in which way these results should be included in the model predictions. All the available microscopic models of MEC depart from the (local) Fermi Gas model. Purely combinatoric arguments (number of possible p-n and n-n pairs) lead to different conclusions than SRC arguments (p-n pair is six or more times more likely than n-n). In the case of SM model (as it is also for the full MEChM model) an extra complication comes from the three nucleons fi-

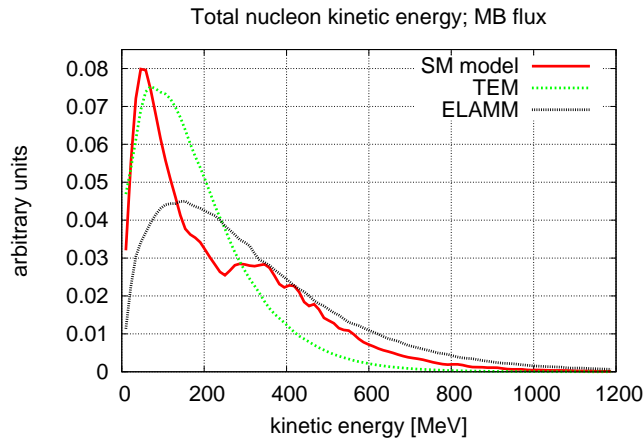


FIG. 18. Distribution of total ejected nucleons kinetic energy averaged over MB flux.

nal states for which it is even more difficult to make any assumptions. Altogether we think as reasonable to expect that in CC neutrino reactions $\sim 80\%$ of final states nucleons are protons and only $\sim 20\%$ are neutrons. For CC antineutrino reactions the isospin composition of the final state is the opposite with $\sim 80\%$ of neutrons and $\sim 20\%$ of protons.

TEM gives us no hints about final nucleons isospin states and we would like to propose the same rules as those assumed for the microscopic SM model.

C. Flux integrated predictions

We would like also to show some flux averaged predictions. In the case of SM model results should be treated with a caution because the model is basically a non-relativistic one (even if some relativistic corrections are included) and neutrino fluxes in experiments like MB, T2K, MINERvA extend to the energies at which relativistic effects are important. The minimal consistency requirement that we checked is that at large energies behaviour of the SM is stable.

Figs 18 and 19 show predictions from three models (including ELAMM) for the total kinetic and maximal nucleon momentum integrated over the MB ν_μ flux. As expected, the differences are quite large and they can be understood by the previous discussion.

VI. CONCLUSIONS

In the paper a multinucleus ejection model is proposed which can be easily implemented in neutrino MC event generators and can help in selecting true CCQE events. The model proposed in the Section III can be used in conjunction with any model providing predictions for the charge lepton inclusive neutrino-nucleus cross sec-

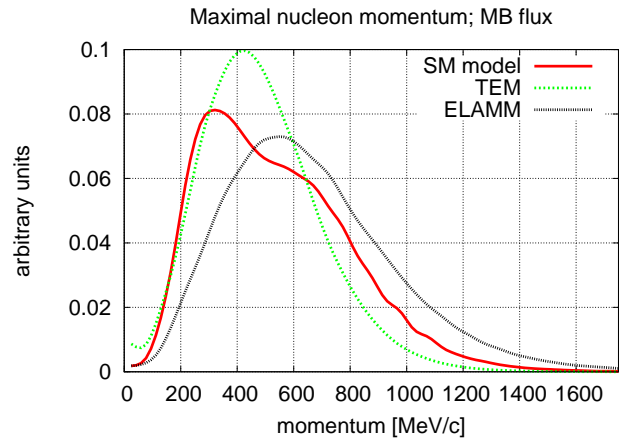


FIG. 19. Distribution of most energetic nucleon momentum averaged over MB flux.

tion. The models of muon inclusive cross sections discussed in the paper are only approximations and it can be shown that they do not reproduce MB double differential cross section too well²¹; the problem is that there are no other available (in the form applicable to MC simulations) models to predict muon inclusive cross section due to MEC mechanism. Our model is very simple but in reality, there are also FSI effects which must be included and which would smear out many details of a better justified multinucleon emission model.

The quantitative understanding of the size and kinematical characteristics of the multinucleon ejection contribution can only come from coincidence charged lepton and final hadronic system measurements in several exclusive channels. This is clearly a very demanding goal.

The observable which in the minimal degree depends on the details of a multinucleon ejection model is the integrated kinetic energy deposit of nucleons. In order to compare with the model predictions for individual nucleon momenta a detector with a low threshold for reconstruction proton tracks is a preferable choice. The liquid argon target with the threshold as low as ~ 300 MeV/c seems to be the best one. 800 MeV neutrinos can produce a sizable fraction of MEC events with two reconstructed proton tracks.

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